CHARMS: A Simple Framework for Adaptive Simulation

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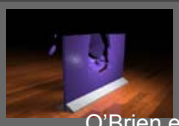
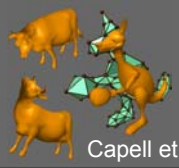
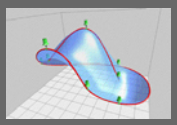
Context: Graphics & PDEs


Debunne et al.

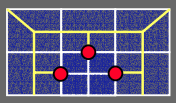

O'Brien et al.


Capell et al.


Welch et al., Gortler et al.

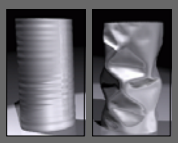
Adaptivity Is Critical

Adjust spatial resolution
- element refinement
  - Cohen, Hodgins,...



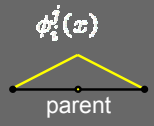
Difficult...
- compatibility
- t-vertices, cracks

...or unfitted!
- subdiv., wavelets,...



Overview

Universal approach to adaptivity
- indep. of: error estimator/solver
- foundation: refinement relation

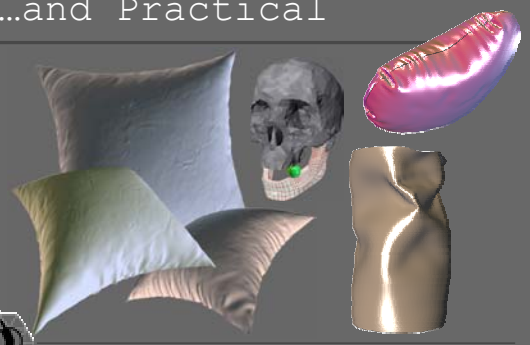


Idea Takeaway

Conforming Hierarchical Adaptive Refinement Methods

Element refinement considered harmful
- refine basis functions, not elements
- simple (naturally compatible!)
- any dimension, order, element type

...and Practical



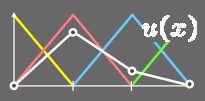
Setting

Example problem

- Laplace's eqn. $\Delta u(x) = 0$

FE Discretization

- $u(x)$ built of basis functions

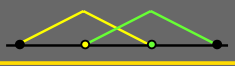


- linear system $\mathbf{Ku} = \mathbf{b}$
  - fn. overlaps $k_{ij} = a\langle \phi_i, \phi_j \rangle$

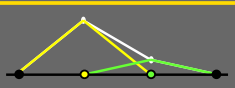
Discretization

Basis view

- linear comb. of functions



$$u(x) = \sum_i u_i \phi_i(x)$$



Discretization

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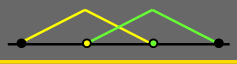

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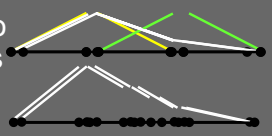
Element view

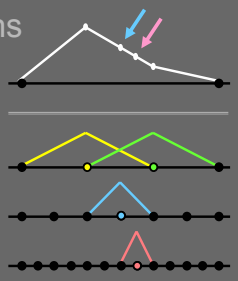
- restriction onto finite elements
- to refine: split



Basis Refinement

Add finer functions

- hierarchy
- linear comb. across levels

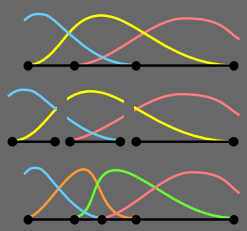


$$u(x) = \sum_{i,j} u_i^j \phi_i^j(x)$$

Element View: Trouble

Higher order schemes

- b-splines, etc.
  - ~~knot insertion~~
  - bivariate case?



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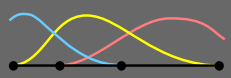
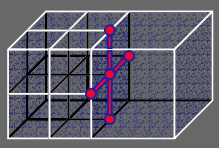


2D, 3D, N-D

- incompatibilities
- cumbersome

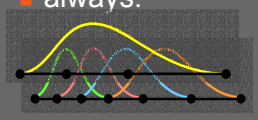


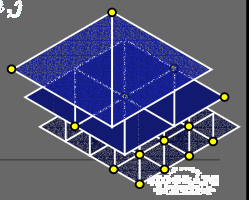
Basis View: Easy

N-D, high order: business as usual

- naturally compatible
- always:

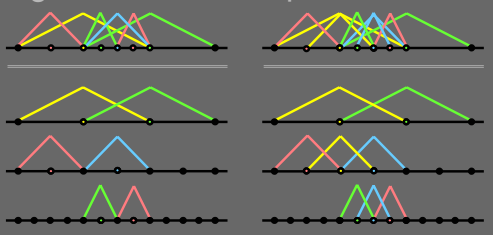
$$u(x) = \sum_{i,j} u_i^j \phi_i^j(x)$$



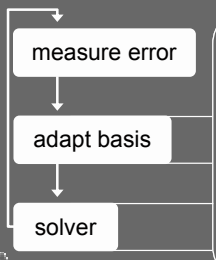


Refinement Strategies

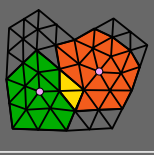
Augment Replace



Framework



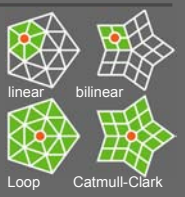
- manage quadratures
  - tesselation
  - function overlap



Definitions

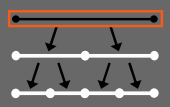
Natural support $S(\phi)$

- set of overlapping same-level elements



Descendants $D(e)$

- set of overlapping finer-level elements



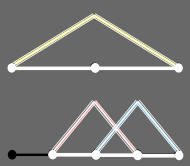
Data Structures

Data structures with invariants

- active functions $\mathcal{B}$
- active elements $\mathcal{E}$

$$\mathcal{E} \equiv \bigcup_{\phi \in \mathcal{B}} S(\phi)$$

- local overlap tables
  - which functions overlap this element?



Stiffness Matrix

To build stiffness matrix:

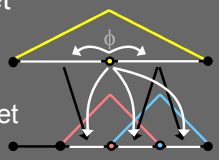
- on every active element:
  - evaluate operator on function pairs
- solver sees only matrix and DOFs

$$k_{ij} = a\left\langle \phi_i, \phi_j \right\rangle$$

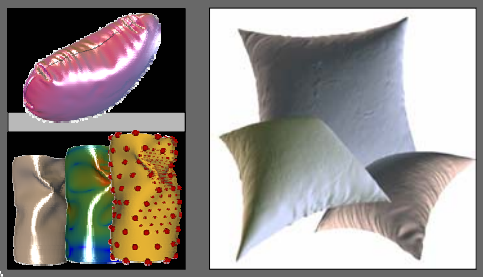
Activation

To activate function ϕ:

- add ϕ to active set
- for $e \in S(\phi)$
  - activate
  - update overlap set
  - for $d \in D(e)$
    - update overlap set



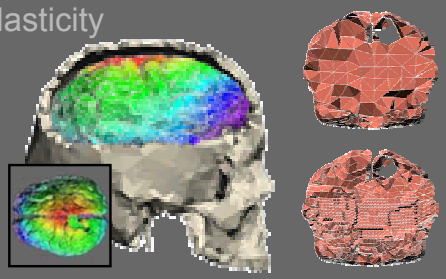
Thin-Shells



Thin-Shells [MOVIE]

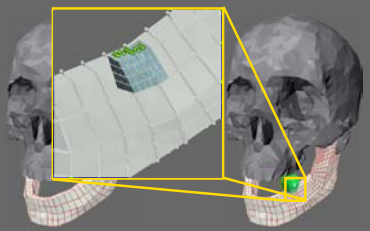
Surgery Simulation

Elasticity



Biomechanics

Hexahedral elements



Conclusion

Adaptivity is easy!

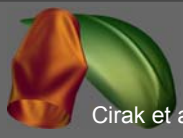
- think basis functions, not elements
- naturally compatible
- debug in 1D: easy port to 2D, 3D

Future

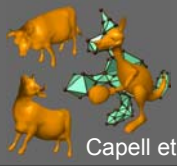
- fluids, surgery simulation
- error criteria, solvers

Context: Graphics & PDEs

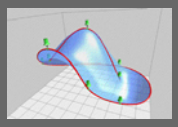
Debunne et al.



Cirak et al.



Capell et al.



Welch et al., Gortler et al.